SIMULTANEITY IN SPECIAL RELATIVITY - 2

by

Ardeshir Mehta

Ottawa, Canada
Thursday, October 25, 2001

RELATIVITY claims that if two events are simultaneous in an inertial frame of reference, then they cannot be simultaneous in *another* inertial frame of reference which is moving uniformly and rectilinearly at a velocity **v** relative to the first inertial frame of reference.

But if the Lorentz transformation equations are correct, this claim results in a clear contradiction, as follows:

1. Let there be an inertial frame of reference — which we shall designate as **I** — in which there are two clocks $\mathbf{C_1}$ and $\mathbf{C_2}$, separated from one another spatially, and *synchronised:* so that whenever the clock $\mathbf{C_1}$ indicates a moment in time $\mathbf{t_1}$, the other indicates a moment in time $\mathbf{t_2}$ such that $\mathbf{t_1 = t_2 = t}$.

2. Let there be *another* inertial frame of reference — which we shall designate as **I'** — also moving rectilinearly and uniformly at a velocity **v** relative to **I**, in which there are two more spatially separated clocks $\mathbf{C'_1}$ and $\mathbf{C'_2}$, which are *also* synchronised: that is, whenever the clock $\mathbf{C'_1}$ indicates a moment **t'** coinciding with the moment **t** indicated by the clock $\mathbf{C_1}$, the clock $\mathbf{C'_2}$ also indicates the *same* moment **t'** indicated by the clock $\mathbf{C'_1}$.

3. Now when the clock $\mathbf{C_1}$ indicates any particular moment $\mathbf{t_1}$, the moment $\mathbf{t_1}$ must be related to the moment $\mathbf{t'_1}$ indicated by the clock $\mathbf{C'_1}$ by the Lorentz transformation equation

   $$\mathbf{t'_1 = (t-vx/c^2)/(1-v^2/c^2)^{0.5}}$$

   ... where **x** is the distance, as measured by an observer in **I**, between clock $\mathbf{C_1}$ and clock $\mathbf{C'_1}$, and the moment $\mathbf{t = t_1}$ is that indicated by the clock $\mathbf{C_1}$ ... and **c** is of course the speed of light.

4. So when $\mathbf{C_1}$ and $\mathbf{C_2}$ both indicate a particular moment $\mathbf{t = t_1 = t_2}$, $\mathbf{C'_1}$ indicates a particular moment $\mathbf{t'_1}$.

5. And when the clock $\mathbf{C_2}$ indicates the same moment $\mathbf{t_2 = t = t_1}$ as is indicated by Clock $\mathbf{C_1}$, the moment $\mathbf{t_2}$ must be related to the moment $\mathbf{t'_2}$ indicated by the clock $\mathbf{C'_2}$ by the Lorentz transformation equation

   $$\mathbf{t'_2 = (t-vy/c^2)/(1-v^2/c^2)^{0.5}}$$

   ... where **y** is the distance, as measured by an observer in **I**, between clock $\mathbf{C_2}$ and clock $\mathbf{C'_2}$, and the moment **t** is, again, that indicated by the clock $\mathbf{C_2}$ ... and **c** is again the speed of light.

6. So when the clocks $\mathbf{C_1}$ and $\mathbf{C_2}$ *both* simultaneously indicate a particular moment $\mathbf{t = t_1 = t_2}$, the clock $\mathbf{C'_2}$ indicates a particular moment $\mathbf{t'_2}$.

7. Now unless $\mathbf{x = y}$ above — which is highly unlikely, though of course not impossible — it is clear that $\mathbf{t'_1}$ will *not* be equal to $\mathbf{t'_2}$ above — both of them being indicated, of course, when both the clocks $\mathbf{C_1}$ and $\mathbf{C_2}$ indicate the *single* moment in time $\mathbf{t = t_1 = t_2}$.

8. But this contradicts Point No. 2. above, according to which whenever $\mathbf{C'_1}$ indicates any particular moment $\mathbf{t'}$, $\mathbf{C'_2}$ must *also* indicate the *same* moment $\mathbf{t'}$, since both $\mathbf{C'_1}$ and $\mathbf{C'_2}$ are *synchronised.*

Or expressed in table form:

ACCORDING TO POINT NO. 2, WHEN:

$\mathbf{C_1}$ indicates	$\mathbf{C_2}$ indicates	$\mathbf{C'_1}$ indicates	$\mathbf{C'_2}$ indicates	Such that
$\mathbf{t_1 = t_2 = t}$	$\mathbf{t_2 = t_1 = t}$	$\mathbf{t'_1}$	$\mathbf{t'_2}$	$\mathbf{t'_1 = t'_2}$

ACCORDING TO POINTS NOS. 3, 5 AND 7, WHEN:

$\mathbf{C_1}$ indicates	$\mathbf{C_2}$ indicates	$\mathbf{C'_1}$ indicates	$\mathbf{C'_2}$ indicates	Such that
$\mathbf{t_1 = t_2 = t}$	$\mathbf{t_2 = t_1 = t}$	$\mathbf{t'_1}$	$\mathbf{t'_2}$	$\mathbf{t'_1 \neq t'_2}$ *

It is of course abundantly clear that the above two tables contradict one another in their fifth columns.

Any comments? e-mail me.

* Or at least, not necessarily.