MATHEMATICAL CONTRADICTIONS ARISING FROM THE LORENTZ TRANSFORMATION EQUATIONS

by

Ardeshir Mehta

Ottawa, Canada
Wednesday, November 28, 2001

IT CAN easily be shown that when the Lorentz transformation equations are applied — and even applied *correctly* — contradictions can arise. One such contradiction is brought to light below.

Suppose that a primed inertial frame of reference is moving relative to an un-primed inertial frame of reference at velocity **v** in the **x** direction. Then according to the Lorentz transformations, the space and time co-ordinates **t**, **x**, **y**, **z** of an event in the un-primed frame are related to the space and time co-ordinates **t'**, **x'**, **y'**, **z'** of the same event in the primed frame by the following equations (when the units are expressed in terms in which speed of light, **c**, is unity):

$$t = \gamma t' + \gamma v x'$$
$$x = \gamma v t' + \gamma x'$$
$$y = y'$$
$$z = z'$$

… where $\gamma = 1/(1 - v^2)^{0.5}$.

(See for example <http://casa.colorado.edu/~ajsh/sr/construction.html>.)

Now let's suppose we have two identical and accurate clocks **C1'** and **C2'** in the primed frame. Suppose they are both located in a *very* long spaceship: a spaceship of "proper length" 10 light-seconds. (This means it would take light 10 seconds, as measured by a clock carried on board the *spaceship,* to reach its tail end from its nose end.) Suppose that the spaceship is moving past a *third* clock **C**, in a direction parallel to the line between the two clocks **C1'** and **C2'**, at a velocity $v = 0.75^{0.5}$ **c** = 0.86602540378444 **c**.

Suppose all the clocks are accurate to one part in a billion: that is, they gain or lose one second in a billion seconds — which, by the way, is almost 32 years. In other words, we're talking about quite a high level of accuracy here.

And additionally, suppose that the readouts of the clocks are all the way up to the microsecond level. That is, the clocks read not only hours, minutes and seconds, but thousandths and even millionths of a second. Only if the two clocks both agree to the microsecond do they truly agree.

Let's suppose that we perfectly synchronise both clocks **C1'** and **C2'** at the front end of the spaceship, and then we move clock **C2'** slowly but steadily away from clock **C1'** towards the rear end of the spaceship at a speed **u** equal to one hundred-millionth (*i.e.,* 10^{-8}) of the speed of light. Since the speed of light is 299,792,458 m/s, one hundred-millionth of that is just a touch less than 3 m/s, or about 10.8 km/h: in ordinary terms, not much faster than a brisk walk.

At that speed it will take, of course, a *billion* seconds — almost 32 years — for clock **C2'** to reach the rear end of the spaceship, which as we said is a billion light-seconds away (as measured in the spaceship's frame). And let's suppose that once clock **C2'** gets to its destination, it comes to a stop relative to the spaceship.

So now let's see whether clock **C2'** could have lost any time at all relative to clock **C1'** during this long journey, due to so-called Relativistic time dilation. The formula for time dilation is:

$$\Delta \mathbf{t} = \gamma \Delta \mathbf{t_o}$$

where:

$$\gamma = \mathbf{1/(1 - u^2)^{0.5}},$$

$\Delta \mathbf{t_o}$ is the "proper time" indicated by **C1'**: that is, the time interval which would be measured by an observer at rest with respect to **C1'**, and

$\Delta \mathbf{t}$ is the "co-ordinate time": that is the time interval which would measured by an observer at rest with respect to **C2'**.

(See <http://aci.mta.ca/Courses/Physics/4701_97/EText/TimeDilation.html>.)

So γ here would be $1/[1 - (10^{-8})^2]^{0.5}$ which is $1/(1 - 10^{-16})^{0.5}$. Now of course $(1 - 10^{-16})$ is 0.999 999 999 999 999 900 000 exactly, and so $(1 - 10^{-16})^{0.5}$ is 0.999 999 999 999 999 950 000 (accurate to 21 decimal places), and thus $1/(1 - 10^{-16})^{0.5}$ is 1.000 000 000 000 000 050 000. So the difference between this and 1.00 exactly is only about 5 parts in 10^{17}.

So, since $\Delta \mathbf{t_o}$ = a billion seconds = 10^9 seconds, $\Delta \mathbf{t} = 10^9 (1 - 5\ \ 10^{-17}) = (10^9 - 5\ \ 10^{-8})$ seconds.

In other words, an observer at rest with respect to clock **C2'** would see clock **C1'** lose time by an amount equal to about *five hundred-millionths* of a second — *i.e.,* five hundredths of a microsecond. Since each of the clocks are only accurate to one part in a billion, and it takes one billion seconds for clock **C2'** to reach the rear end of the spaceship from the front end, suffering much *less* than one whole second of time loss in the process, this basically means that time dilation would *not occur at all* within the limits of accuracy of the clocks. It wouldn't even register on them, in fact, since their readouts are only capable of displaying a microsecond difference.

So now we have *two perfectly synchronised clocks* **C1'** and **C2'** in the spaceship, one at either end. Because of their great accuracy, they could read at most only ± 1 second different from one another.

Since the “proper length” of the spaceship is 10 light-seconds, the distance between the two clocks **C1'** and **C2'** in the frame of the spaceship — *i.e.,* in the primed frame — is 10 light-seconds. In other words, if we were to assume clock **C1'** to be at primed co-ordinate $\mathbf{x_1'} = 0.00$, then clock **C2'** would be at primed co-ordinate $\mathbf{x_2'}$ = 10 light-seconds (*i.e.,* 2,997,924,580 m.)

Now since the velocity **v** between clock **C** and the spaceship is $0.75^{0.5}\ \mathbf{c} = 0.86602540378444\ \mathbf{c}$, $\gamma = 1/(1 - 0.75/1^2)^{0.5} = 1/0.25^{0.5} = 1/0.5 = 2.00$. And according to the equations quoted above, the time co-ordinate **t** of an event in the un-primed frame should be related to the time co-ordinate **t'** of the same event in the primed frame by the equation $\mathbf{t} = \gamma\mathbf{t'} + \gamma\mathbf{vx'}$.

Consider the set of events in which clock **C** is just a few centimetres away from clock **C1'** while the spaceship is whizzing by clock **C** at velocity $\mathbf{v} = 0.75^{0.5}\ \mathbf{c} = 0.86602540378444\ \mathbf{c}$ at an instant when clock **C1'** is reading **t'**. In this set we have the following three events:

[1] Clock **C1'** reading **t'** ± 1 sec. at time co-ordinate **t'** and space co-ordinate $\mathbf{x_1'}$ in the primed frame, where $\mathbf{x_1'} = 0.00$,

[2] Clock **C2'** reading **t'** ± 1 sec. at time co-ordinate **t'** and space co-ordinate $\mathbf{x_2'}$ in the primed frame where $\mathbf{x_2'}$ = 10 light-seconds (*i.e.,* 2,997,924,580 m), and

[3] Clock **C** reading **t** at time co-ordinate **t** and space co-ordinate **x** in the un-primed frame.

(Remember that by the accuracy limits of the clocks, clock **C2'** must be reading less than *one second* different from the readout of clock **C1'**. Thus as far as the two clocks' *readouts* are concerned, the time co-ordinates of both the events [1] and [2] above must be **t'** ± 1 sec.)

Now let’s calculate, using the Lorentz transformation equations, what the time co-ordinate of clock **C** should be, using the time co-ordinates of each of the clocks **C1'** and **C2'** in turn.

Applying the equation $\mathbf{t} = \gamma\mathbf{t'} + \gamma\mathbf{vx'}$ to calculate the time co-ordinate of clock **C**, using the figures from [1] and [3] above, we get:

(A) $\mathbf{t} = 2\mathbf{t'} + (2.00 \quad 0.75^{0.5}\ \mathbf{c} \quad 0 \text{ light-seconds}) = 2\mathbf{t'} + 0 = 2\mathbf{t'}$;

… while applying the same equation to calculate the time co-ordinate of clock **C**, using the figures from [2] and [3] above, we get:

(B) $\mathbf{t} = 2\mathbf{t'} + (2.00 \quad 0.75^{0.5}\ \mathbf{c} \quad 10 \text{ light-seconds}) = (2\mathbf{t'} + 17.32 \ldots)$ seconds.

In other words, if the above-quoted Lorentz transformation equations of Relativity are correct, clock **C** must be reading *both* 2**t'** ± 1 sec. *and* (2**t'** + 17.32 …) seconds ± 1 sec. — but of course each of these “correct” solutions contradicts the other!

Any comments? e-mail me.